Lecture 11
THE EQUATION OF SHEARING FORCE FOR THE BEAMS

Plan

1. The classification of beams.
2. About statically determinate and statically indeterminate beams.
3. The strength - weight ratio under torsion.

11.1. The classification of beams.

A bar subject to forces or couples that lie in a plane containing the longitudinal axis of the bar is called a ***beam***. The forces are understood to act perpendicular to the longitudinal axis.

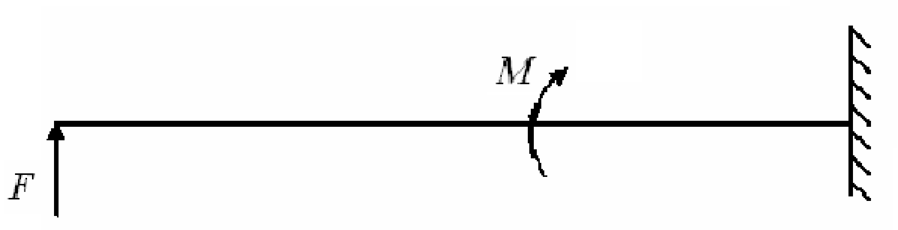


Fig. 11.1.

If a beam is supported at only one end and in such a manner that the axis or the beam cannot rotate at that point, it is called **a *cantilever beam***. This type of beam is illustrated in Fig. 11.1. The left end of the bar is free to deflect but the right end is rigidly clamped. The right end is usually said to be "restrained". The reaction of the supporting wall at the right upon the beam consists of a vertical force together with a couple acting in the plane of the applied loads shown.

A beam that is freely supported at both ends is called a ***simple beam***. The term freely "supported" implies that the end supports are capable of exerting only forces upon the bar and are not capable of exerting any moments. Thus, there is no restraint offered to the angular rotation of the ends of the bar at the supports as the bar deflects under the loads. Two simple beams are sketched in Fig. 11.2.

It is to be observed that at least one of the supports must be capable of undergoing horizontal movement so that no force will exist in the direction of the axis of the beam. If neither end were free to move horizontally, then some axial force would arise in the beam as it deforms under load.

The beam of Fig. 11.2, a is said to be subject to a concentrated force; that of Fig. 11.2, b is loaded by a uniformly distributed load as well as a couple.

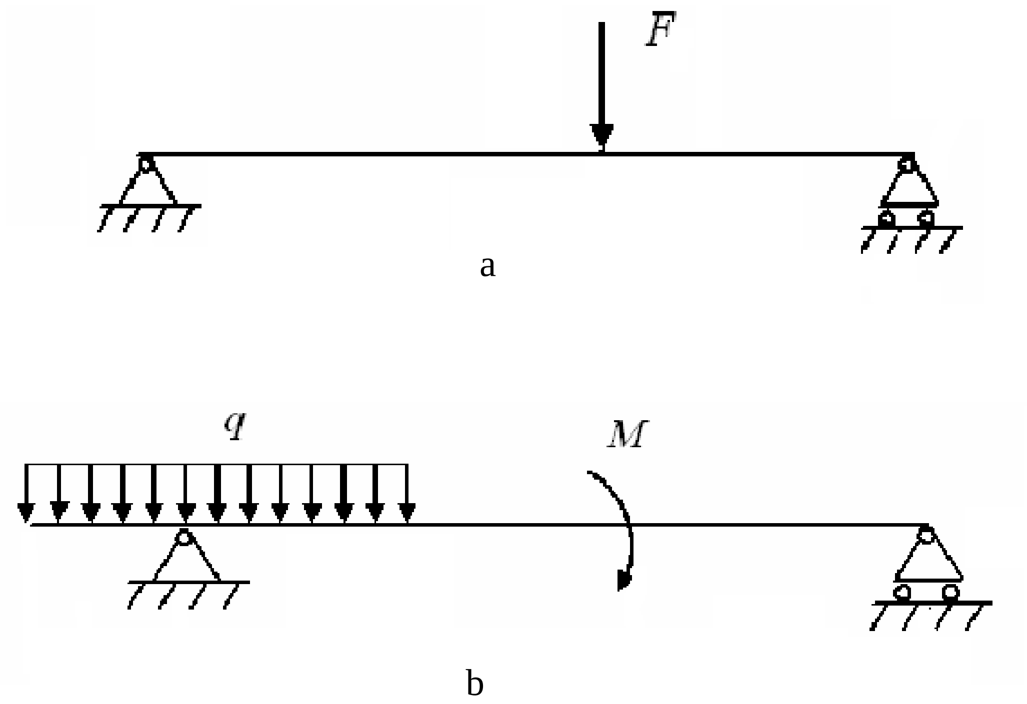


Fig. 11.2.

A beam freely supported at two points and having one or both ends extending beyond these supports is termed an ***overhanging beam***. Two examples are given in Fig. 11.3.

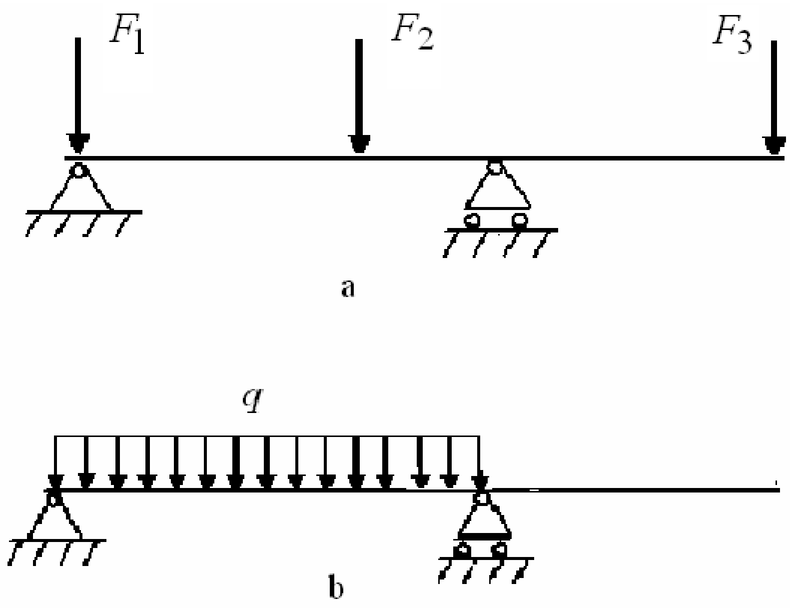


Fig. 11.3

On beginning

11.2. About statically determinate and statically indeterminate beams

All the beams considered above, the cantilevers, simple beams, and overhanging beams, are ones in which the reactions of the supports may be determined by use of the equations of static equilibrium. The values of these reactions are independent of the deformations of the beam. Such beams are said to be ***statically determinate***.

If the number of reactions exerted upon the beam exceeds the number of equations of static equilibrium, then the static equations must be supplemented by equations based upon the deformation s of the beam. In this case the beam is said to it ***statically indeterminate***. Example is shown in Fig. 11.4.

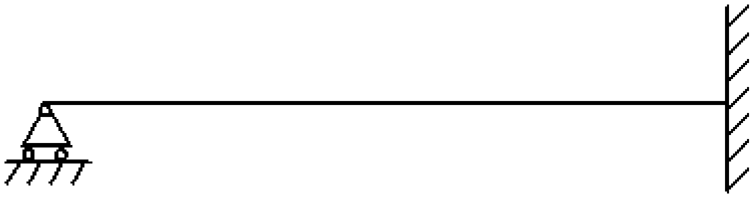


Fig. 11.4.

Loads commonly applied lo a beam may consist of concentrated forces (applied at a point). uniformly distributed loads, in which case the magnitude is expressed as a certain number of pounds per fool or Newtons per meter of length of the beam, or uniformly varying loads. This last type of load is exemplified in Fig. 11.5.

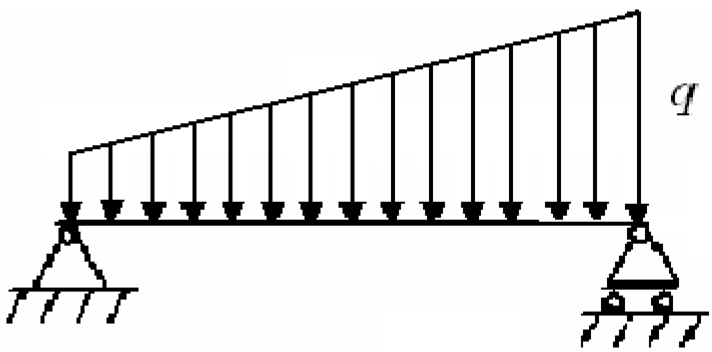


Fig. 11.5.

A beam may also be loaded by an applied couple. The magnitude of the couple is usually expressed in lb·ft or N·m.

When a beam is loaded by forces and couples, internal stresses arise in the bar. In general, both normal and shearing stresses will occur. In order to determine the magnitude of these stresses al any section of the beam, it is necessary to know the resultant force ·and moment acting at that section. These may be found by applying the equations of static equilibrium.

On beginning

11.3. About connection of shearing forces and bending moments.

Suppose several concentrated forces act on a simple beam as in Fig.11.6, a.

It is desired to study the internal stresses across the section at D located a distance x from the left end of the beam. To do this let us consider the beam to be cut at D and the portion of the beam to the right of D removed. The portion removed must then be replaced by the effect it exerted upon the portion to the left of $D$and this effect will consist of a vertical shearing force together with a couple, as represented by the vectors Q and M, respectively, in the free - body diagram of the left portion of the beam shown in Fig. 11.6, c.

The force Q and the couple M hold the left portion of the bar in equilibrium under the action of the forces R_1, P_1, P_2. The quantities Q and M are taken to be positive if they have the senses indicated above.

The couple M shown in Fig. 11.6, c is called the resisting moment at section D. The magnitude of M may be found by use of a static equation which states that the sum of the moments of all forces about an axis through D and perpendicular to the plane of the page is zero. Thus,

$$\sum M_o = M - R_1 \cdot x + P_1(x-a) + P_2(x-b) = 0,$$

or

$$M = R_1 \cdot x - P_1(x-a) - P_2(x-b).$$

Thus, the resisting moment M is the moment at point D created by the moments of the reaction at A and the applied forces P_1 and

P_2. The resisting moment M is the resultant couple due to stresses that are distributed over the vertical section at D. These stresses act in a horizontal direction and are tensile in certain portions of the cross section and compressive in others.

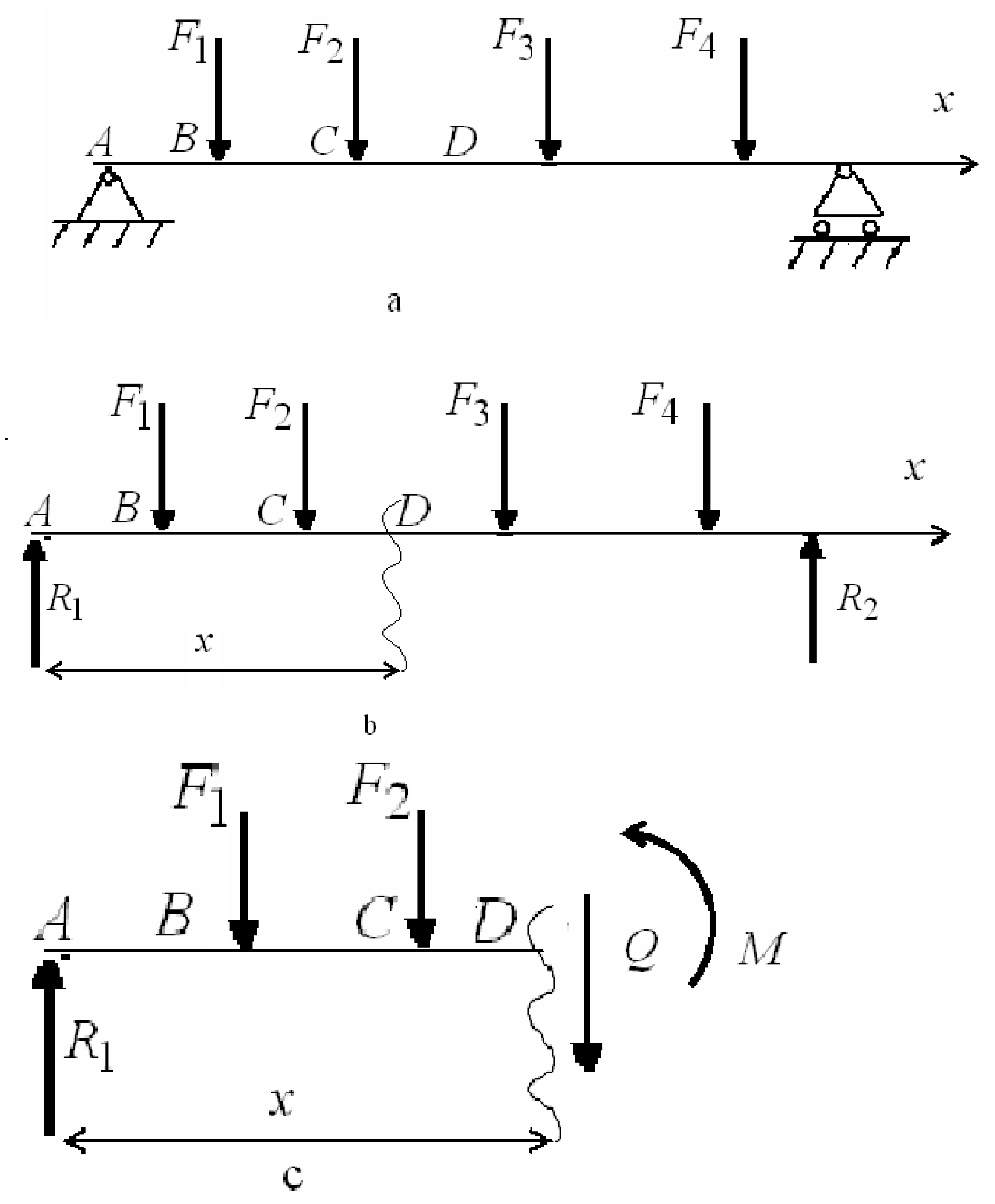


Fig. 11.6.

The vertical force Q shown in Fig.11.6, c is called the resisting shear at section D. For equilibrium of forces in the vertical direction,

$$\sum F_y = R_1 - P_1 - P_2 - Q = 0,$$

or

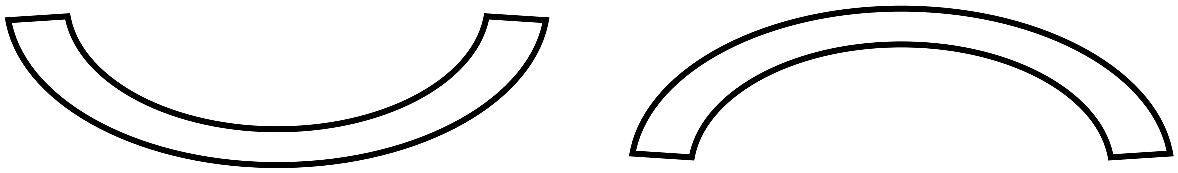
$$Q = R_1 - P_1 - P_2.$$

This force Q is actually the resultant of shearing stresses distributed over the vertical section at D.

The algebraic sum of the moments of the external forces to one side of the section D about an axis through D is called the bending moment at D. This is represented by

$$R_1 \cdot x + P_1(x-a) + P_2(x-b)$$

for the loading considered above. Thus, the bending moment is opposite in direction to the resisting moment but is of the same magnitude. It is usually denoted by M also. Ordinarily, the bending moment rather than the resisting moment is used in calculations because it can be represented directly in terms of the external loads.



Positive bending   Negative bending

Fig. 11.7.

The algebraic sum of all the vertical forces to one side, say the left side, of section D is called the shearing force at that section. This is represented by $R_1 - P_1 - P_2$ for the above loading. The shearing force is opposite in direction to the resisting shear but of the same magnitude. Usually it is denoted by Q. It is ordinarily used in calculations, rather than the resisting shear.

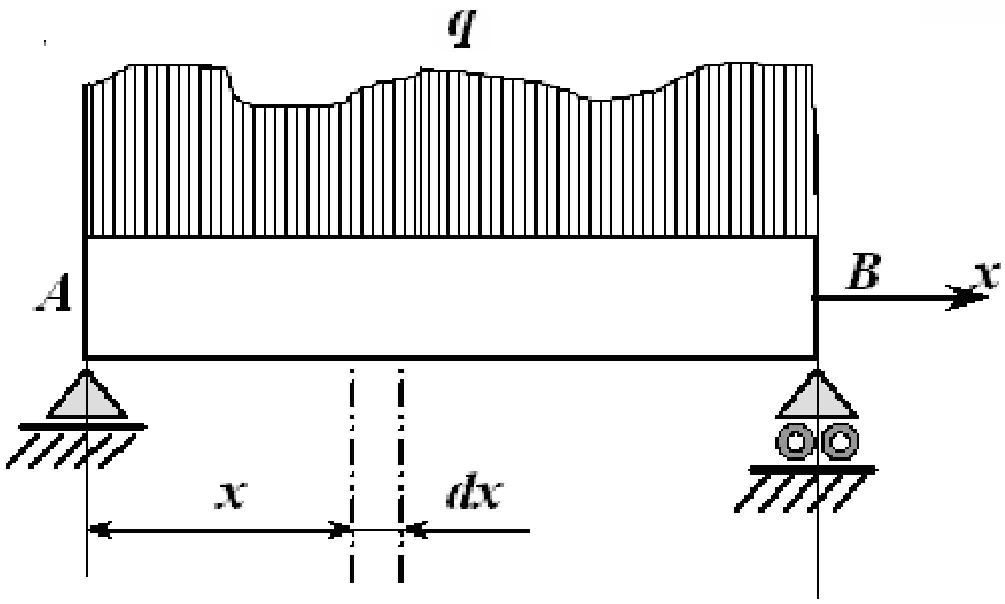


Fig. 11.8.

The customary sign conventions for shearing force and bending moment are represented in Fig. 11.7. Thus, a force that tends to bend the beam so that it is concave upward is said to produce a positive bending moment. A force that tends to shear the left portion of the beam upward with respect to the right portion is said to produce a positive shearing force.

An easier method for determining the algebraic sign of the bending moment at any section is to say that upward external forces produce positive bending moments, downward forces yield negative bending moments.

A simple beam with a varying load indicated by $q(x)$ is sketched in Fig. 11.8. The coordinate system with origin at the left end A is established and distances to various sections in the beam arc denoted by the variable x.

For any value of x the relationship between the load $q(x)$ and the shearing force Q is:

$$q = \frac{dQ}{dx}, \tag{11.1}$$

and the relationship between shearing force and bending moment M is:

$$Q = \frac{dM}{dx} . \qquad (11.2)$$

For ease in treating problems involving concentrated forces and concentrated moments we introduce the function:

$$f_n(x) = (x - a)^n ,$$

where for $n > 0$ the quantity in pointed brackets is zero if $x < a$ and is the usual $(x - a)^n$ if $x > a$. This is the singularity or half - range function. Thus, if the argument is positive the pointed brackets behave just as ordinary parentheses.

On beginning